Depolarization of opinions on social networks through random nudges

Ritam Pal,^{1,*} Aanjaneya Kumar,^{1,†} and M. S. Santhanam^{1,‡}

Polarization of opinions has been empirically noted in many online social network platforms. Traditional models of opinion dynamics, based on statistical physics principles, do not account for the emergence of polarization and echo chambers in online network platforms. A recently introduced opinion dynamics model that incorporates the homophily factor – the tendency of agents to connect with those holding similar opinions as their own – captures polarization and echo chamber effects. In this work, we provide a non-intrusive framework for mildly nudging agents in an online community to form random connections. This is shown to lead to significant depolarization of opinions and decrease the echo chamber effects. Remarkably, even a mild nudge is seen to be effective in avoiding polarization, though a large nudge leads to another undesirable effect, namely, radicalization. Further, we obtain the optimal nudge factor to avoid the extremes of polarization and radicalization outcomes.

I. INTRODUCTION

The information revolution [1–3] has lowered the entry barrier for nearly everyone to participate and contribute towards shaping opinions [4–6] and even policies on a wide gamut of issues. This has been largely aided by the easy availability of social media infrastructure through mobile devices. Increasingly, the collective opinions expressed through various social media platforms are thought to be one barometer of the public mood on any contentious issue of the day. This provides an interesting testing ground for the dynamics, and statistical physics of interacting multi-agent systems since the online nature of interactions provides fine-grained data for quantitative analysis and comparison with model results. [7–13]

The analysis of opinion dynamics from the perspective of statistical physics can be traced back to early works in which a parallel has been drawn between an ensemble of interacting spins – Ising model [14] – and mutual influence exerted by agents among one another [8, 15, 16]. The voter model [9, 17, 18] introduced in 1975 also has a strong basis in a framework of interacting spins on a regular lattice but provides a simple set of rules for how the preferences of agents change with time. In the original voter model, each agent (spin) modifies its own opinion based on that of a randomly chosen neighbor. These quantitative approaches suggest that large participatory interactions among agents might lead to the emergence of consensus [19–21]. However, empirical results have shown that the distribution of opinions among real people tends to show a bimodal distribution pattern, especially on controversial issues of the day [22-26]. Over the years, the main aim behind the variants of the basic models has been to capture these observational trends. Recent reviews on this topic are available in Refs. [18, 27]

Another empirical feature that could not be accounted

for by these early models (at least by their original version) was the phenomenon of echo chambers [28]. This refers to a scenario in which one agent's opinion is similar to that of the agents in their "social neighborhood," and one tends to reinforce the other. Lack of sufficient engagement with opposing opinions leads to positive reinforcement of one's own opinion within the circle of a close-knit social network. Empirical evidence for this effect has been reported from several social media platforms [28, 29]. This is also known to be responsible for sustaining misinformation for a longer time on social networks [30, 31].

Recently, a simple model of opinion dynamics [7] was proposed to account for the observed features from empirical data. The two features often encountered in polarized engagements on social networks are (a) most active users tend to be strongly polarized, and (b) locally connected agents on a network tend to have a convergence of opinions. Both these are shown to arise through a mechanism of reinforcement of opinion among the agents and the tendency of the agents to interact more with those with similar opinions (homophily [7, 10, 32]). Even if the model starts from an initial distribution of opinions without clear preferences, the interactions induce the formation of polarized states.

It might appear that in the case of controversial and polarising topics, the interaction and the debate will always lead to polarized states of opinion. This outcome leaves no room for the reconciliation of disparate opinions. Contrary to what we might expect based on reallife experiences, it might lead to a rather unfortunate inference that interaction is the cause of polarization. In this work, we show that if a small number of agents are nudged, then the cycle of reinforcement of opinions can be broken, and depolarization can be achieved. In the context of social networks, the nudges are effected by exposing a small fraction polarized agents to differing opinions. We also show that overdoing this leads to radicalization [33, 34], a state in which all the agents hold the same opinion. We formulate the optimization problem that avoids both polarization and radicalization and compute the right amount of nudge required for achieving

¹Department of Physics, Indian Institute of Science Education and Research, Dr. Homi Bhabha Road, Pune 411008, India.

^{*} ritam.pal@students.iiserpune.ac.in

 $^{^{\}dagger}$ kumar.
aanjaneya@students.iiserpune.ac.in

[‡] santh@iiserpune.ac.in

this optimal scenario.

In the next section, we discuss the basic model and motivate the random nudges in the subsequent section. In Sec.IV, we demonstrate our results and discuss their implications. We formulate an optimization problem, in Sec.V, which emerges from a trade-off between the depolarization due to the proposed random nudges and the tendency to move towards a radicalized state. We conclude with a discussion of future directions.

II. BASIC MODEL AND METHODS

To analyze polarization and to introduce possible intervention methods for reducing polarization, we adapt the recently introduced model for opinion dynamics [7]. This model qualitatively captures a few aspects of opinion dynamics when agents' opinions evolve due to interactions in social media platforms. The model is able to reproduce the empirical features such as polarization and echo chambers and the fact that more active people on social media tend to have extreme opinions.

The model has N interacting agents, and it is assumed that there are only two possible sides to an opinion. This is typical of many, but not all, the issues – for example, to allow abortion or not. Opinion on a given issue is denoted by x_i , which can take any real value in the range $(-\infty,\infty)$. The sign of the x_i corresponds to the stance of the agent in the corresponding issue, and the $|x_i|$ denotes the conviction of the agent in their respective stance. This implies that the larger the value of $|x_i|$, the more extreme the opinion of the agent is. The model used to capture the evolution of opinion is activity driven [35–37], i.e., at each time step, only active agents are allowed to interact with other agents. Based on empirical data [36, 38], the probability for agents to be active is chosen to be

$$F(a) = \frac{1 - \gamma}{1 - \varepsilon^{1 - \gamma}} a^{-\gamma},\tag{1}$$

where a is the activity, ε is the minimum activity (chosen in this work to be 10^{-2}), and γ controls how steep the function F(a) which is chosen to be $\gamma = 2.1$. Agents' opinions evolve based on their interactions with other agents, and this information is encoded in the time-dependent adjacency matrix $A_{i,j}(t)$. Further, opinion evolution also depends on the strength of social interaction K > 0 and the controversialness of the issue $\alpha > 0$. The opinion dynamics is given by the following N coupled differential equations [7]

$$\dot{x}_i = -x_i + K \left(\sum_{j=1}^N A_{ij}(t) \tanh\left(\alpha x_j\right) \right). \tag{2}$$

In this, $A_{i,j}(t)$ is the temporal adjacency matrix of interaction at time t. If there is an input from agent j to i at time t, than $A_{i,j}(t) = 1$, and $A_{i,j} = 0$ otherwise.

If agent i is active at time t, they will interact with m other agents, weighted by the probability $P_{i,j}$. Further, the probabilistic reciprocity factor $r \in [0,1]$ determines the chance that an interaction is mutually influential, i.e., $A_{ij}(t) = A_{ji}(t) = 1$. The interaction probability is defined to be a function of the magnitude between two agents' opinions.

$$P_{ij} = \frac{|x_i - x_j|^{-\beta}}{\sum_j |x_i - x_j|^{-\beta}},$$
 (3)

where β is the homophily factor and quantifies the tendency for agents with similar opinions to interact with each other: $\beta=0$ refers to the absence of interaction preference, and $\beta>0$ implies that the agents with similar opinions are more likely to interact with one another. Evidently, Eq. 3 is modeled as a power-law decay of connection probabilities with only a small chance for agents with opposite opinions to interact. Since most of the interactions tend to occur between agents with similar opinions, this can lead to the formation of echo chambers.

The interaction dynamics in the model is enforced by the activity-driven temporal network that is fully encoded by the parameters $(\varepsilon, \gamma, m, \beta, r)$, together with the parameters that characterises the issue, (K, α) . Asymptotically, this model features three distinct states in the distribution of opinions. If the Social interaction K is sufficiently small, then the opinion of every agent decays to zero, and this state is known as the consensus state. However, if social interaction K is large but the homophily factor β is small, then due to statistical fluctuations, all the opinions either become positive or negative. This is the state of radicalization. And the most interesting case emerges when social interaction K and homophily factor β are large enough. In this case, the opinion distribution shows a polarization state.

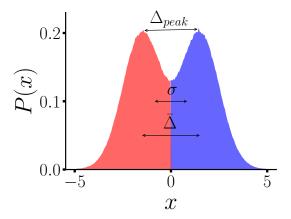


FIG. 1. A schematic to illustrate three measures of polarization. $\bar{\Delta}$ is the distance between mean positive and negative opinions. Δ_{peak} denotes the distance between two peaks in the opinion distribution, and σ denotes the standard deviation of the opinion distribution.

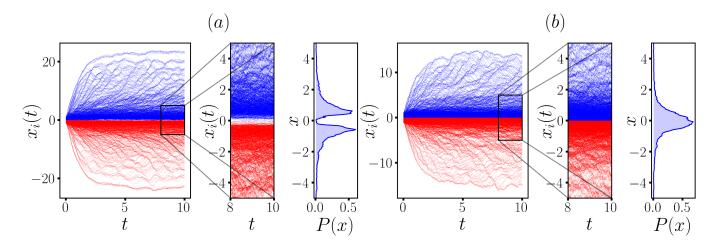


FIG. 2. Emergent polarization (and depolarization) states in the presence (and absence) of the nudge factor. The simulations are performed with parameters set to be in the polarization regime. (a) The agents are not nudged. Hence the polarized state emerges. A magnification of the region around x=0 reveals the absence of trajectories there, and the corresponding distribution shows a bimodal distribution. (b) Network nudge is introduced with probability p=0.01, and we find a significant depolarization. Opinion trajectories tend to crowd around x=0, and a unimodal distribution emerges.

III. RANDOM NUDGES AND POLARIZATION

Echo chambers are increasingly becoming more apparent in online social media platforms. A generic tendency to interact with people who hold similar opinions as ours can lead to echo chambers, and this effect is, in turn, amplified by the recommendation engines on social media platforms. These algorithmically driven engines recommend similar connections or content in order to keep the users of those platforms engaged [39]. These two features are modeled by the homophily interaction probability β . Large values of β represent how closed the echo chambers are. To disrupt the formation of echo chambers even while keeping the platforms as engaging as possible and without violating the users' privacy, we adopt the following intervention in the opinion dynamics model: with probability p < 1, the active agents will interact uniformly with any other agents, and with probability, (1-p) the active agents will interact with others according to the homophily probability given in Eq. 3. We call p random network nudge. As p does not depend on the opinions of the agents, the intervention is noninvasive (the recommendation engine need not interpret the opinion of the agents), and for small enough values of p, it is hoped that the platform is still engaging while maintaining enough diversity to ensure there is no echo chamber. With this intervention, we propose a modified interaction probability as

$$\widetilde{P}_{ij} = p \times \frac{1}{N-1} + (1-p) \times P_{ij}. \tag{4}$$

This is used in the rest of the results shown in this paper.

Defining Polarization: Before we discuss the results, we discuss the three quantities employed to measure the degree of polarization based on the opinion distribution

g(x). They are defined as: (a) Polarization is measured through $\bar{\Delta}$, defined as the distance between the average of positive opinions and the average of negative opinions. This is shown in the schematic in Fig. 1. (b) Polarization can also be measured by the distance between the maxima in the distribution of positive and negative opinions, denoted by Δ_{peak} [40]. This is particularly useful when the distribution clearly exhibits a bimodal character (See Fig. 1. (c) A gross measure of polarization could be the standard deviation σ of the entire opinion distribution [39], as indicated in Fig. 1. It must be noted that if polarization decreases as a result of the intervention proposed in Eq. 4, then all these three quantifiers must decrease.

IV. RESULTS

With the intervention strategy introduced in Sec. III, we find that with sufficiently small random nudge probability p, we obtain significant depolarization in the opinion distributions characterized by a unimodal distribution along with the decay of all three measures of polarization. To see the effects of nudge, we perform numerical simulations of the basic model in Eq. 2 using the interaction probability given in Eq. 3 and the intervention model in Eq. 4. The simulations are performed with N=10000 agents for t=1000 time steps with dt=0.01. At initial time x_i is uniformly chosen from a small interval, i.e., $x_i \in [-1,1]$ for i=1,2...N. The model parameters are chosen to be $\alpha=3$, $\beta=3$, K=3, m=10, $\gamma=2.1$, $\varepsilon=0.01$ and r=0.5 for all the simulations.

In Fig. 2, we choose parameters corresponding to the asymptotic polarized state. It shows the contrast between the trajectories of individual opinions and the

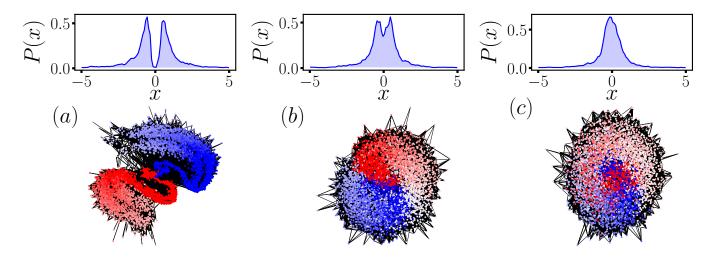


FIG. 3. Effect of the nudge on the opinion distribution and the structure of social interactions networks. The networks are averaged over the last 100 time steps of simulation and are drawn using the draw function in networkx [41]. Nodes with blue color correspond to agents with positive opinions, and red corresponds to agents with negative opinions. The saturation of the color is mapped to the conviction of the agents; high saturation corresponds to a high level of conviction, and vice-versa. (a) For p = 0, i.e., without a nudge, the distribution is polarized, and the network has two distinct clusters, one formed by the agents with positive opinions and the other by the agents with negative opinions. (b) For p = 0.005, the opinion distribution is significantly depolarized, and the network has more connections between the positive and negative opinionated clusters of nodes. (c) For p = 0.01, we observe a unimodal distribution of opinion, and the social interactions network is now well mixed. A depolarization state is reached.

opinion distribution without and with the application of a nudge. In the absence of nudge (p=0), the simulation results in Fig. 2(a) show less trajectories with opinions $x_i \approx 0$. This leads to a bimodal distribution of opinions characteristic of a polarized state. In contrast, in Fig 2(b), a small nudge probability of p=0.01 is applied, and we find significantly many more trajectories with moderate opinions. This, effectively, is seen to lead to an absence of polarization characterized by a unimodal distribution. The magnifications of the region around $x_i = 0$ and its distribution (shown in Fig. 2) reveal a clear contrast between these two scenarios.

To examine the effect of network nudge, we analyze the underlying time-averaged structures of the temporal interactions network. Without nudge, the (timedependent) interaction network has two distinct clusters; most of the connections are among positive opinionated agents or negative opinionated agents. There exist very few connections between these two groups other than for the agents with extreme opinions. This is expected since the agents with extreme opinions are also those who tend to be more active on social networks fora; hence on average, they form more connections. This enables them to be relatively more connected to the agents with opposing opinions. These results are visually depicted in Fig. 3 as three snapshots of evolving network diagrams. If p = 0, no nudge is applied. In this case, as Fig 3(a) shows, a polarized network made up of two distinct blue and red-colored clusters is visible. Blue color corresponds to nodes with x > 0, and red color to x < 0. The opinion

distribution shown at the top confirms the existence of polarization.

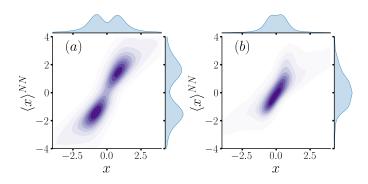


FIG. 4. Echo chamber effect with and without the application of random nudge. The opinion of an agent x and the mean opinion of its nearest neighbors averaged over 20 realizations are plotted (see Eq. 5). (a) Nudge not applied (p=0). The presence of two lobes is indicative of the echo chamber effect. (b) Nudge applied (p=0.01). A single lobe shows the weakening of the echo chamber effect.

However, when a nudge is applied, even for the case when the nudge probability is as small as p=0.005, there are still many more connections within the positively opinionated group (blue) and negative opinionated group (red), but signs of depolarization are visible in the network diagram (Fig. 3(b)). It is confirmed by opinion distribution which now only shows a shallow dip in the vicinity of x=0. For p=0.01 displayed in Fig. 3(c),

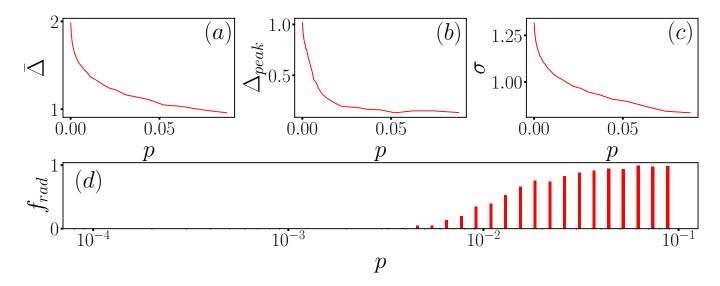


FIG. 5. Three measures of polarization, (a) $\bar{\Delta}$, (b) Δ_{peak} , and (c) σ , as a function of nudge strength p. All three polarization parameters are averaged over the last 100 time steps and also averaged over 200 realizations. (d) shows the fraction of simulations that lead to radicalization for different nudge strengths.

we find the network to be well mixed (large blue and red clusters have disappeared), and this leads to a greater degree of depolarization.

The term echo chamber describes a situation where the beliefs or opinions of people are reinforced by interactions among a closed group of people who hold similar opinions. In recent years, this has been widely discussed in the context of online communities [28, 42], though some studies appear to suggest that the effects of echo chambers are over-estimated [43]. To infer the presence of echo chamber-type effects, we calculate the average opinion of the nearest neighbors (NN) of each agent [44]. This is denoted by

$$\langle x \rangle^{NN} = k_i^{-1} \sum_j a_{ij} x_j, \quad \text{and} \quad k_i = \sum_j a_{ij}, \quad (5)$$

where a_{ij} is the temporally aggregated adjacency matrix. When a nudge is not applied (p=0), a colored contour plot of x and $\langle x \rangle^{NN}$, in Fig. 4(a) reveals two saturated spots corresponding to the two distinct echo chambers. The parameters used for this simulation are the same as in Fig. 3(a). Now, when we apply a nudge with probability p=0.01, we can observe only one saturated spot indicating the existence of only one closed group. All the agents are inside this closed group, and the echo chamber effect is largely diluted or non-existent. This is more clearly evident in the distributions plotted along the top and right-side axes of Fig. 4.

V. OPTIMIZING THE NUDGE: POLARIZATION VERSUS RADICALIZATION

To obtain a global picture of how depolarization sets in as a function of nudge probability p, we plot the three

measures of polarization as a function of p. All three measures, Δ, Δ_{peak} and σ , have been computed from the simulation results. The results shown represent an average over the last 100 time steps of simulation and averaged over 200 realizations. In Fig. 5, we observe that all three measures of polarization decrease as the strength of the nudge p increases. In particular, $\bar{\Delta}$ and σ are found to decrease as a stretched exponential function $\exp(-p^{\gamma})$, and the stretching factor γ is determined through regression to be approximately 0.3. A recent work studying the depolarization of echo-chambers [40] considered adding an effective noise term dependent on a random sample of opinions to Eq. (2). While this approach succeeds in making the opinion distribution unimodal, it increases the width of the distribution significantly, which as a consequence corresponds to an increase in extreme opinions. In contrast, the framework of nudging the mechanism of forming social connections in online interactions works well in decreasing extreme opinions, and also suggests direct algorithmic interventions for recommender systems.

One limitation of the intervention proposed in this work is that for large p, we observe that in a large fraction of the realizations, a radicalized state emerges. This can be seen in Fig. 5(d), which displays the fraction of realizations that lead to radicalization f_{rad} as a function of p. It is clear that radicalization is either non-existent or a rarity for $p < 10^{-2}$, while radicalization becomes the norm for $p > 10^{-2}$. In many situations, radicalization is as much undesirable as polarization.

To solve the issue of radicalization at a high value of nudge strength, rather than nudging all the people in the population, we nudged a fraction of the population. We define a simple linear utility function $U(\bar{\Delta}, f_{rad}) = \tilde{\Delta} + f_{rad}$ Where $\tilde{\Delta}$ is $\bar{\Delta}$ linearly scaled to be between 0 and 1, and f_{rad} is the fraction of radicalized simulations.

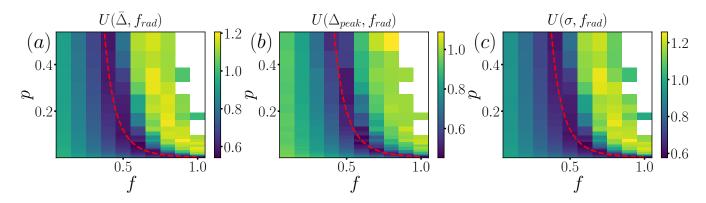


FIG. 6. The heat map of the utility as a function of nudge strength and population fraction. Panel (a), (b), and (c) corresponds to the corresponding utility of $\bar{\Delta}$, Δ_{peak} , and σ , respectively. The red dashed curve, which is found to follow the curve $p \cdot f^A = B$, (A, B = constants), demotes the optimal values of population fraction and nudge strength.

The structure of the utility function is the same for the other two measures of polarization. In Fig. 6 we find the optimal values of population fraction and nudge strength, which follows the curve $p \cdot f^A = B$ (A, B = constants).

VI. DISCUSSION

The widespread use of the internet, and consequently, social media platforms, has drastically altered the way humans consume and interact with information. polarization and the formation of echo chambers have been shown to negatively impact constructive discussions and debates – two fundamental pillars of a healthy democracy. Building on the recent advances in the modeling of opinion dynamics in social networks, in this work we study the possibility of depolarizing a population using a stochastic nudge.

Our results suggest that a small number of randomized interactions, which are other dominated by homophily-driven mechanisms, can lead to a significant reduction in polarization. This reduction was quantitatively captured by three different measures of polarization. While we show that minimal nudges can burst echo chambers and lead to socially desirable distributions of opinions, increasing the strength of this nudge can result in radicalization. Given this sensitivity on the nudge strength, we show that a possible resolution is obtained if, instead of nudging each agent, only a fraction f of the agents are

nudged. We highlight that this interplay of the nudge strength p and the fraction f of nudged individuals leads to an interesting optimization problem. This optimization can help inform the fraction of individuals to be nudged for a fixed nudge strength for optimal depolarization.

We believe that the strongest case for the application of such randomized nudges can be made to recommendation systems. While ubiquitous, recommender algorithms are optimized for increasing engagement, which we now know can come at the cost of creating echo chambers, increase in the representation of extreme ideologies, and even the tampering of users' preferences. In such settings, the randomized nudges can be potentially operationalized as the poisoning of a viewer's watch history with a limited amount of random content, uncorrelated with the viewer's preferences. While there are several ethical and legal considerations that must be accounted for before implementing any such interventions, it certainly opens up several interesting avenues for future research to build on.

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